Maneuver Strategies for Multiplanet Missions

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Theme

THIS paper compares a number of maneuver strategies for five multiplanet missions, including the single planet strategy of nulling the aim-plane and flight-time errors at the immediate target. Significant differences were found among the strategies in the amount of corrective propellant required and the residual miss at each target. In general, nulling or minimization of the residual miss at the next target was found to be superior to that at the immediate target, when error growth is not nulled early. The best strategy corrects aim-plane and flight-time errors at the next target.

Contents

The success of a multiplanet mission depends upon the guidance success at each of the targets, accomplished by one or more impulsive maneuvers. Each maneuver has only three degrees of freedom. A trajectory, however, has six degrees of freedom. Therefore, considerable choice exists in the maneuver criterion (or strategy). The effectiveness of a particular strategy, under the additional consideration of propellant economy, is the subject of this study.

For a single planet mission, the choice of $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$, and T_f (the vector components of closest approach distance and the associated time) as a terminal set is adequate for guidance success.[1] However, when subsequent encounters are planned, stringent accuracy is implied on the B-plane miss (in the nominal plane of motion) and the asymptote direction at each of the intermediate targets. Correcting only the B-plane and T_f errors would result in asymptote direction errors, which would cause B-plane, T_f, and asymptote direction errors at the subsequent targets. Correcting only the $\mathbf{S}\cdot\mathbf{R}$, $\mathbf{S}\cdot\mathbf{T}$ and C_3 errors (the direction and magnitude of the incoming hyperbolic velocity) would have similar results. Moreover, the correctional capability of a maneuver is limited because a three-component incremental velocity cannot null the errors in a six-component terminal set. This conflict necessitates investigation of alternate strategies.

The investigation of alternate strategies is complex because the correctional capability is time varying. For an early maneuver, the changes are mainly in $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$, and T_f, whereas, a late maneuver changes all six components. What strategy should be used for a maneuver at any particular time? This paper attempts to answer this question by considering several strategies employing alternate parameters as a terminal set.

Additional parameters for consideration are $\mathbf{S}\cdot\mathbf{R}$, $\mathbf{S}\cdot\mathbf{T}$, C_3, hyperbolic excess velocity vector, etc. The analysis considers a number of three-component and six-component vectors as candidate terminal sets. For a 3-vector terminal set, the velocity correction vector is uniquely determined. However, for a 6-vector terminal set, the velocity correction vector is obtained by simultaneous minimization of the propellant and the residual miss. These considerations lead to a number of different maneuver strategies.

The problem considered here is: Given a nominal multiplanet trajectory, all required mapping matrices, perfect knowledge of the trajectory, perfect maneuver execution, and assumed perturbation in velocity, $\mathbf{e}$, at a fixed time t_1, then find a $\Delta\mathbf{v}$ at time $t_2(\geqq t_1)$ so as to minimize the terminal errors.

Let

$$\mathbf{m} = [\mathbf{B}\cdot\mathbf{R}\,\mathbf{B}\cdot\mathbf{T}\,T_f\,\mathbf{S}\cdot\mathbf{R}\,\mathbf{S}\cdot\mathbf{T}\,C_3]^T \quad (1)$$

$$\mathbf{v} = [\dot{x}\dot{y}\dot{z}]^T \quad (2)$$

and let $\mathbf{p}$ and $\mathbf{q}$ be the incoming and outgoing hyperbolic excess velocity vectors, respectively. Let the subscripts J and $J+1$, respectively, denote the immediate and the next target. The following mapping matrices are available over $[t_1, t_J]$ where t_J is the nominal time of encounter of target J:

$$\partial\mathbf{m}_J/\partial\mathbf{v}(t), \partial\mathbf{m}_{J+1}/\partial\mathbf{v}(t), \partial\mathbf{p}_J/\partial\mathbf{v}(t), \partial\mathbf{p}_{J+1}/\partial\mathbf{v}(t), \partial\mathbf{q}_J/\partial\mathbf{v}(t), \partial\mathbf{q}_{J+1}/\partial\mathbf{v}(t)$$

It can be shown that, for each strategy, $\Delta\mathbf{v} = F\mathbf{e}$, where F is expressible in terms of the mapping matrices. Then, the resulting residual miss vector is given by

$$\Delta^r\mathbf{m}_i = [\partial\mathbf{m}_i/\partial\mathbf{v}(t_1) + F\cdot\partial\mathbf{m}_i/\partial\mathbf{v}(t_2)]\mathbf{e} \quad (3)$$

and the weighted residual miss vector is defined by

$$\Delta_w{}^r\mathbf{m}_i = W_i\Delta^r\mathbf{m}_i \quad (4)$$

where $i = J, J+1$; and W_i are weighting matrices. The ten strategies are described in Table 1.

The relative merit of the strategies depends upon: 1) mission; 2) times t_1 and t_2 and the associated mapping matrices; and 3) velocity perturbation, $\mathbf{e}$. All of these factors were considered.

The results presented are representative and pertain to an Earth-Jupiter-Saturn mission; t_1 is 10 days after Earth departure, t_2 is varied continuously; and, $\mathbf{e} = [0.1\ \ 0.1\ \ 0.1]^T$ m/sec. Normalized time is defined by

$$t = (t_2 - t_1)/(t_J - t_1) \quad (5)$$

In order to assess the effect of delays in t_2 and provide a means for ranking the strategies, two loss functions were selected. Each

Table 1 Description of strategies

Strategy no.	Description
1	Nulling of $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$ and T_f errors at the immediate target
2	Nulling of $\mathbf{S}\cdot\mathbf{R}$, $\mathbf{S}\cdot\mathbf{T}$ and C_3 errors at the immediate target
3	Minimization of $\|\Delta^r\mathbf{m}_J\|$
4	Minimization of $\|\Delta_w{}^r\mathbf{m}_J\|$
5	Nulling of $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$, and T_f errors at the next target
6	Nulling of $\mathbf{S}\cdot\mathbf{R}$, $\mathbf{S}\cdot\mathbf{T}$, and C_3 errors at the next target
7	Minimization of $\|\Delta^r\mathbf{m}_{J+1}\|$
8	Minimization of $\|\Delta_w{}^r\mathbf{m}_{J+1}\|$
9	Nulling of $\mathbf{q}$ errors at the immediate target
10	Nulling of $\mathbf{p}$ errors at the immediate target

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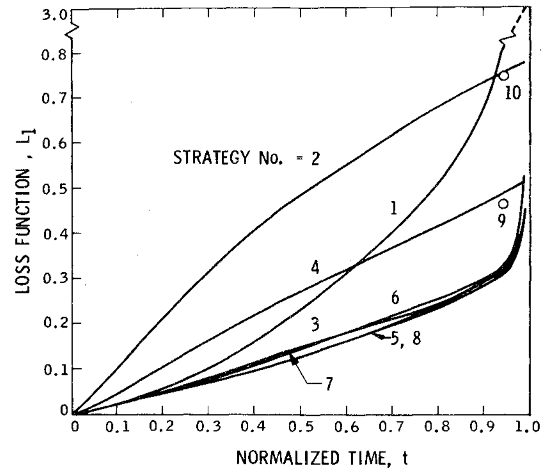

Fig. 1 Loss function, L_1.

function was suitable as a measure of the goodness of the strategies. These are

$$L_1(t) = \|\Delta \mathbf{v}\|/d_1 + \|\Delta_w{}^r\mathbf{m}_J\|/d_2 + \|\Delta_w{}^r\mathbf{m}_{J+1}\|/d_3 \tag{6}$$

where d's are chosen such that the supremum of each term in Eq. (6) for all strategies and all t is 1; and

$$L_2(t) = \log_{10}[1 + \|\Delta\mathbf{v}\| \cdot \|\Delta_w{}^r\mathbf{m}_J\| \cdot \|\Delta_w{}^r\mathbf{m}_{J+1}\|] \tag{7}$$

Figure 1 shows the plots of L_1 as a function of t. For almost all maneuver times, strategies 3, 5, 6, 7, and 8 appear superior. Strategies 5 and 8 are indistinguishable and are the best. All

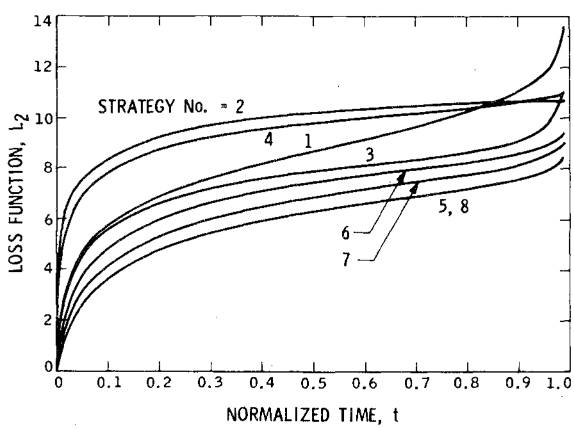

Fig. 2 Loss function, L_2.

Table 2 Ranking of strategies

Rank	Strategy no.	Description
I	5, 8	Nulling of $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$, and T_f errors at the next target; minimization of the norm of the weighted errors in miss vector (of six components) at the next target
II	7, 6, 3	Minimization of the norm of the unweighted errors in miss vector (of six components) at the next target; nulling of $\mathbf{S}\cdot\mathbf{R}$, $\mathbf{S}\cdot\mathbf{T}$, and C_3 errors at the next target; minimization of the norm of the unweighted errors in miss vector at the immediate target
III	9, 4, 1	Nulling of $\mathbf{q}$ errors at the immediate target; minimization of the norm of the weighted errors in miss vector at the immediate target; nulling at the immediate target of $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$, and T_f errors
IV	2, 10	Nulling at the immediate target of $\mathbf{S}\cdot\mathbf{R}$, $\mathbf{S}\cdot\mathbf{T}$, and C_3 errors; nulling of $\mathbf{p}$ errors at the immediate target

strategies show an increase in the loss function as the maneuver is delayed. Strategy 1 shows the maximum increase, followed by strategies 2, 3, and 4. Figure 2 shows plots of L_2 and accentuates strategy differences. General observations are essentially the same. Over-all strategy ranking can be accomplished by integrating the loss function.

General conclusions are: 1) The single-target strategy of nulling $\mathbf{B}\cdot\mathbf{R}$, $\mathbf{B}\cdot\mathbf{T}$, and T_f errors at the immediate target is the least economical in propellant, since it produces large asymptote ($\mathbf{S}\cdot\mathbf{R}$, $\mathbf{S}\cdot\mathbf{T}$, and C_3) errors when trajectory dispersions are not corrected early. 2) A similar strategy for the next target is the best and produces the same results as minimizing the norm of the weighted residual miss vector (of six components) at the next target. 3) In general, any strategy based on minimizing errors at the next target is superior to a strategy accomplishing the same at the immediate target. 4) If only a 3-component error vector is to be considered whose nulling at the immediate target produces a reasonable strategy, then the choice is the $\mathbf{q}$ errors (from among the strategies considered). 5) There is a considerable dispersion among the strategies in the amount of propellant used and the residual miss at each target. The amount of dispersion depends on the type of perturbation, proximity of the maneuver to the encounter, and mission. However, in all cases, the following group-wise ranking of the strategies (Table 2) remains unchanged.

Reference

[1] Kizner, W., "A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories," JPL External Publications 674, Aug. 1, 1959, Jet Propulsion Lab., Pasadena, Calif.